ALGEBRA

1. Daraja (butun ko'rsatkichli)

Bir xil ifodalarning ko'paytmasiga daraja deyiladi. $2\cdot2\cdot2\cdot2\cdot2 = 2^5$; $x\cdot x\cdot x = x^3$; $a\cdot a\cdot a \ldots a = a^n$ $(a \neq 0,\ n\in N)$

a — asos, n — daraja ko'rsatkich, a^n — daraja.

Daraja bilan berilgan amalda:

1) $a^0=1$, 2) $a^1 = a$ $(a \neq 0)$, 3) $a^{2n} > 0$ $(a \neq 0)$,

4) $a^{-n} = \dfrac{1}{a^n}$ $(a \neq 0)$, 5) $(-a)^{2n} = a^{2n}$, 6) $(-a)^{2n+1} = -a^{2n+1}$,

7) $a^n\cdot a^k = a^{n+k}$, 8) $a^n : a^k = a^{n-k}$, 9) $(a^n)^k = a^{nk}$, 10) $(ab)^n = a^n b^n$,

11) $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$ $(b \neq 0)$, 12) $\left(\dfrac{a^n}{b^m}\right)^{-k} = \left(\dfrac{b^m}{a^n}\right)^k$ $(a \neq 0,\ b \neq 0)$.

2. Ildiz
(kasr ko'rsatkichli daraja)

n — darajasi $(n\in N)$ a ga teng bo'lgan b son (ifoda), a ning n — darajali ildizi deyiladi $(n\geq2)$.

$\sqrt[n]{a} = b$, agar $b^n = a$.

$$\sqrt[n]{a^n} = \begin{cases} |a|, & n = 2k,\quad k \in N, \\ a, & n = 2k+1,\quad k \in N. \end{cases}$$

$n = 2$ da $\sqrt{a}$ — 2-darajali (kvadrat) ildiz.

Kasr ko'rsatkichli darajada:

1) $\sqrt[n]{a^k} = a^{\frac{k}{n}}$,   2) $(\sqrt[n]{a})^n = a \ (a \ge 0)$,

3) $\sqrt[mn]{a^{mk}} = \sqrt[n]{a^k}$,   4) $\sqrt[n]{a^k \cdot b^k} = \sqrt[n]{a^k} \cdot \sqrt[n]{b^k}$,

5) $\sqrt[n]{\dfrac{a^k}{b^m}} = \dfrac{\sqrt[n]{a^k}}{\sqrt[n]{b^m}} \ (b \ne 0)$,   6) $\left(\sqrt[n]{a^m}\right)^k = \sqrt[n]{a^{mk}}$,

7) $\sqrt[n]{a^{n+k}} = a\sqrt[n]{a^k}$,   8) $a^{-\frac{m}{n}} = \dfrac{1}{\sqrt[n]{a^m}}$,

9) $a^m \sqrt[n]{b^k} = \sqrt[n]{a^{mn} \cdot b^k}$,   10) $\sqrt[m]{\sqrt[n]{a^k}} = \sqrt[mn]{a^k}$,

11) $\sqrt[m]{a^k} \cdot \sqrt[n]{b^p} = \sqrt[mn]{a^{nk} \cdot b^{pm}}$,   12) $\sqrt{a^2} = a$,

13) $\sqrt[2n]{(a-b)^{2n}} = a - b, \ (a \ge b)$,

14) $\sqrt[2n]{(a-b)^{2n}} = b - a$ agar $a < b$.

15) Ikki hadning 2-tartibli ildizini soddalashtirishda

$$\sqrt{a + \sqrt{b}} = \sqrt{\frac{a + \sqrt{a^2 + b}}{2}} + \sqrt{\frac{a - \sqrt{a^2 - b}}{2}},$$

$$\sqrt{a - \sqrt{b}} = \sqrt{\frac{a + \sqrt{a^2 + b}}{2}} - \sqrt{\frac{a - \sqrt{a^2 - b}}{2}},$$

tengliklar o'rinli.

3. Maxrajni irratsionallikdan qutqarish

Kasr maxrajini irratsionallikdan qutqarishda, kasr surat va maxrajini, maxrajdagi irratsional ifodaning qo'shmasiga ko'paytirish kerak.

1. $\sqrt[n]{a^k}$ ga qo'shma $\sqrt[n]{a^{n-k}}$ $(n > k,\ a > 0)$.

2. $(\sqrt{a} + \sqrt{b})$ ga qo'shma $(\sqrt{a} - \sqrt{b})$ $(a > 0,\ b > 0,\ a \neq b)$.

3. $(\sqrt{a} - \sqrt{b})$ ga qo'shma $(\sqrt{a} + \sqrt{b})$.

4. $(\sqrt[3]{a} + \sqrt[3]{b})$ va $(\sqrt[3]{a^2} - \sqrt[3]{ab} + \sqrt[3]{b^2})$ o'zaro qo'shma.

5. $(\sqrt[3]{a} - \sqrt[3]{b})$ va $(\sqrt[3]{a^2} + \sqrt[3]{ab} + \sqrt[3]{b^2})$ o'zaro qo'shma.

6. Agar maxrajda $(\sqrt{a} + \sqrt{b} + \sqrt{c})$ bo'lsa, oldin $(\sqrt{a} + \sqrt{b} - \sqrt{c})$ ga ko'paytiramiz. Keyin esa $\left[(a + b - c) - 2\sqrt{ab}\right]$ ga ko'paytiriladi.

4. Qisqa ko'paytirish formulalari

1. $(a + b)^2 = a^2 + 2ab + b^2$.
2. $(a - b)^2 = a^2 - 2ab + b^2$.
3. $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$.
4. $(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$.
5. $(a - b)(a + b) = a^2 - b^2$.
6. $(a - b)(a^2 + ab + b^2) = a^3 - b^3$.
7. $(a + b)(a^2 - ab + b^2) = a^3 + b^3$.
8. $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + ac + bc)$.
9. $a^4 - b^4 = (a - b)(a^3 + a^2b + ab^2 + b^3) = (a - b)(a + b) \times (a^2 + b^2)$.
10. $a^5 - b^5 = (a - b)(a^4 + a^3b + a^2b^2 + ab^3 + b^4)$.
11. $a^5 + b^5 = (a + b)(a^4 - a^3b + a^2b^2 - ab^3 + b^4)$.
12. $a^n - b^n = (a - b)(a^{n-1} + a^{n-2}b + \ldots + ab^{n-2} + b^{n-1})$.
13. $(a + b - c)^2 = a^2 + b^2 + c^2 + 2(ab - ac - bc)$.
14. $(a + b)\cdot(a^{2n-1} - a^{2n-2}b + \ldots + ab^{2n-2} - b^{2n-1}) = a^{2n} - b^{2n}$.
15. $(a - b)\cdot(a^{2n-1} + a^{2n-2}b + \ldots + ab^{2n-2} + b^{2n-1}) = a^{2n} - b^{2n}$.
16. $(a + b)\cdot(a^{2n} - a^{2n-1}b + \ldots - ab^{2n-1} + b^{2n}) = a^{2n+1} + b^{2n+1}$.
17. $(a - b)\cdot(a^{2n} + a^{2n-1}b + \ldots + ab^{2n-1} + b^{2n}) = a^{2n+1} - b^{2n+1}$.

18. $(a+b)^n = a^n + na^{n-1}b + \frac{n(n-1)}{2!}a^{n-2}b^2 + \dots + nab^{n-1} + b^n$.

5. Ketma-ketlik

Ketma-ketlik berilgan deyiladi, agar har bir n da ($n \in N$) shu ifodaning aniq hadini ko'rsatish mumkin bo'lsa, masalan, $a_{n+2} = a_n + a_{n+1}$ regurrent formulada $a_1 = 1$, $a_2 = 3$ bo'lsa, ketma-ketlikning birinchi yetti hadi 1; 3; 4; 7; 11; 18; 29, ...

$$a_1, a_2, \dots, a_n$$

ketma-ketlikda ixtiyoriy n larda $a_{n+1} > a_n$ bo'lsa, ketma-ketlik o'suvchi bo'ladi aks holda kamayuvchi deyiladi.

6. Arifmetik progressiya

$a_{n+1} = a_n + d$, $a_1 = a$ regurrent ($n \in N$, $d \neq 0$) munosabatda aniqlangan ketma-ketlik

$$\div\, a_1, a_2, \dots, a_n$$

arifmetik progressiyani ifoda qiladi.

d — progressiya ayirmasi,

$d > 0$ da progressiya o'suvchi,

$d < 0$ da progressiya kamayuvchi,

n-hadni topish formulasi

$$a_n = a_1 + d\cdot(n-1),\ n \in N$$

Birinchi n ta had yig'indisini topish formulasi

$$S_n = \frac{(a_1 + a_n)\cdot n}{2} \quad \text{yoki} \quad S_n = \left[a_1 + \frac{(n-1)\cdot d}{2}\right]\cdot n$$

arifmetik progressiyada

$$a_{k-1} + a_{k+2} = a_k + a_{k+1} \qquad k \neq 1 \quad k \in N,$$

$$a_k = \frac{a_{k-p} + a_{k+p}}{2} \quad (p < k);\ (k \geq 2)\ k \neq 1 \quad k \in N.$$

16) $\log_a M > \log_a N$ yoki ($\log_a M < \log_a N$) da

$$\begin{cases} a > 1 \text{ da } M > N; \ (M < N) \\ 0 < a < 1 \text{ da } M < N; \ (M > N) \end{cases}$$

17) $$\log_a N \begin{cases} > 0, \ a > 1, \ N > 1 \text{ yoki } 0 < a < 1, \ 0 < N < 1 \text{ da} \\ < 0, \ a > 1, \ 0 < N < 1 \text{ yoki } 0 < a < 1, \ N > 1 \text{ da} \end{cases}$$

18) $$\log_a M - \log_a N \begin{cases} > 0, \text{ agar } a > 1 \text{ va } M > N > 0 \text{ bo‘lsa;} \\ < 0, \text{ agar } 0 < a < 1 \text{ va } M > N > 0 \text{ bo‘lsa;} \end{cases}$$

19) $a > 1$ va $0 < b_1 < b_2$ uchun

$$\frac{\log_a b_1 + \log_a b_2}{2} < \log_a \frac{b_1 + b_2}{2};$$

$0 < a < 1$ va $0 < b_1 < b_2$ uchun

$$\frac{\log_a b_1 + \log_a b_2}{2} > \log_a \frac{b_1 + b_2}{2}.$$

9. Kompleks sonlar

9.1. Kompleks sonning algebraik ko‘rinishi

$$z = a + ib, \ i = \sqrt{-1}, \ \ a, \ b \in R \ (i^2 = -1)$$

$a \neq 0$, $b = 0$ da haqiqiy son;

$a = 0$, $b \neq 0$ da mavhum son;

$a = 0$, $b = 0$ da $z = 0$;

$\text{Re} z = a$ — kompleks sonning haqiqiy qismi;

$\text{Jm} z = b$ — kompleks sonning mavhum qismi;

$\bar{z} = a - ib$ kompleks son z kompleks songa qo‘shma;

$z^* = -a - ib$ kompleks son z kompleks songa qarama-qarshi kompleks son. Bunda

$(z + \bar{z})$ haqiqiy son $(z + z^*)$ nol son;

$z\,z = a^2 + b^2; \quad z \cdot z^* = -(a^2 + b^2);$

Agar $z_1 = a_1 + ib_1$ va $z_2 = a_2 + ib_2$ bo'lsa, u holda:

$z_1 \pm z_2 = (a_1 \pm a_2) + i\,(b_1 \pm b_2); \quad \alpha z_1 = \alpha a_1 + i\alpha b_1;$

$z_1 = z_2$ agar $a_1 = a_2$ va $b_1 = b_2$ bo'lsa.

$$|z_1 + z_2| \le |z_1| + |z_2|; \qquad |z_1 - z_2| \ge |z_1| - |z_2|;$$

$$|z_1 \cdot z_2| = |z_1| \cdot |z_2|; \qquad \left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}; \qquad z_2 \ne 0 \quad |z^n| = |z|^n.$$

9.2. Kompleks sonning trigonometrik shakli

$$z = r(\cos\varphi + i\sin\varphi).$$

Bunda $r = |z| = \sqrt{a^2 + b^2}$ — kompleks sonning moduli,

$\varphi = \arg z = \operatorname{arctg}\frac{b}{a}$ — kompleks son argumenti bo'lib,

$$\varphi = \begin{cases} \operatorname{arctg}\frac{b}{a} & \text{agar, } a > 0,\ b > 0 \text{ bo'lsa,} \\ \pi + \operatorname{arctg}\frac{b}{a} & \text{agar, } a < 0,\ b > 0 \text{ bo'lsa,} \\ -\pi + \operatorname{arctg}\frac{b}{a} & \text{agar, } a < 0,\ b < 0 \text{ bo'lsa,} \\ 2\pi + \operatorname{arctg}\frac{b}{a} & \text{agar, } a > 0,\ b < 0 \text{ bo'lsa,} \\ \frac{\pi}{2} & \text{agar, } a = 0,\ b > 0 \text{ bo'lsa,} \\ -\frac{\pi}{2} & \text{agar, } a = 0,\ b < 0 \text{ bo'lsa,} \\ 0 & \text{agar, } a > 0,\ b = 0 \text{ bo'lsa,} \\ \pi & \text{agar, } a < 0,\ b = 0 \text{ bo'lsa.} \end{cases}$$

Burchak umumiy ko'rinishi

$\operatorname{Arg} z = \arg z + 2k\pi \quad (k = 0, \pm 1, \pm 2, \ldots)$

$|\bar{z}| = |z|; \quad \arg \bar{z} = -\arg z.$

Agar z_1 va z_2 kompleks sonlarga $\bar{z}_1$ va $\bar{z}_2$ mos qo'shma kompleks sonlar bo'lsa,

$$\overline{z_1 + z_2} = \bar{z}_1 + \bar{z}_2; \quad \overline{z_1 \cdot z_2} = \bar{z}_1 \cdot \bar{z}_2$$

$z_1 = r_1(\cos\varphi_1 + i\sin\varphi_1)$ va $z_2 = r_2(\cos\varphi_2 + i\sin\varphi_2)$ kompleks sonlar uchun:

$z_1 \cdot z_2 = r_1 \cdot r_2[\cos(\varphi_1 + \varphi_2) + i\sin(\varphi_1 + \varphi_2)];$

$\dfrac{z_1}{z_2} = \dfrac{r_1}{r_2}[\cos(\varphi_1 - \varphi_2) + i\sin(\varphi_1 - \varphi_2)];$

$z^n = r^n(\cos\varphi + i\sin\varphi);$

$\sqrt[n]{z} = \sqrt[n]{|r|}\left(\cos\dfrac{\varphi + 2k\pi}{n} + i\sin\dfrac{\varphi + 2k\pi}{n}\right);$

$\sqrt[n]{r}$ — arifmetik ildiz; $k = 0, 1, 2, 3, \ldots, n - 1.$

Agar Eylerning

$$e^{i\varphi} = \cos\varphi + i\sin\varphi$$

formulasini hisobga olsak, kompleks sonning

$$z = re^{i\varphi} \qquad (e = 2{,}7182\ldots)$$

ko'rsatkichli formasi kelib chiqadi.

10. Determinant

Ikkinchi tartibli determinant

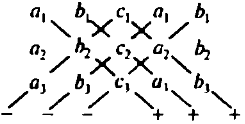
$$\begin{vmatrix} a & b \\ a_1 & b_1 \end{vmatrix} = ab_1 - a_1 b$$

formula bo'yicha hisoblanadi.

Uchinchi tartibli determinant

$$\begin{matrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{matrix} = a_1b_2c_3 + a_3b_1c_2 + a_2b_3c_1 - a_3b_2c_1 - a_1b_3c_2 - a_2b_1c_3$$

formula bo'yicha hisoblanadi. Bu formulani quyidagi Sarrus qoidasi bilan hisoblash mumkin:



11. Birlashmalar

Har qanday narsalardan tuzilgan va bir-biridan shu narsalarning tartibi bilan, yo o'zi bilan farq qiluvchi gruppalar birlashmalar deyiladi.

1. n ta elementdan m tadan ($n, m \in N$, $n \geq m$) o'rinlashtirish deb, shunday birlashmalarga aytiladiki, ularning har birida n elementdan olingan m ta element bo'lib, ular bir-birlaridan yo elementlari bilan, elementlarining tartibi bilan farq qiladi. O'rinlashtirishlar soni

$$A_n^m = n(n-1)(n-2)...(n-m+1) \quad \text{yoki} \quad A_n^m = \frac{n!}{(n-m)!}$$

formuladan topiladi.

Bunda $A_n^0 = 1$; $A_n^{m+1} = (n-m)A_n^m$

$$A_6^3 = 6 \cdot 5 \cdot 4 = 120.$$

2. Faqat elementlarning tartibi bilangina farq qilgan ($n = m$) o'rinlashtirishlar o'rin almashtirishlar deyiladi. O'rin almashtirishlar soni

$$P_n = n! = 1 \cdot 2 \cdot 3 \ldots n$$

formulada topiladi.

Bunda $0! = 1$; $A_n^n = P_n$.

$$P_4 = 1 \cdot 2 \cdot 3 = 24.$$

3. n elementdan m tadan tuzilgan gruppalash deb, n elementdan m tadan tuzilgan o'rinlashtirishlar bir-biridan eng kamida bitta elementi bilan farq qiladigan o'rinlashtirishlarga aytiladi. Gruppalashlar soni

$$C_n^m = \frac{A_n^m}{P_m} = \frac{n(n-1)(n-2)\ldots(n-m+1)}{m!}.$$

Bunda $C_n^0 = C_n^n = 1$; $C_n^1 = C_n^{n-1} = n$; $C_n^m = C_n^{n-m}$; $C_{n-1}^m + C_{n-1}^{m-1} = C_n^m$.

$$C_8^4 = \frac{A_8^4}{P_4} = \frac{8 \cdot 7 \cdot 6 \cdot 5}{1 \cdot 2 \cdot 3 \cdot 4} = 7 \cdot 2 \cdot 5 = 70.$$

12. Nyuton binomi formulasi

Binom so'zi ikki had degan ma'noni bildiradi, faqat ikkinchi hadi bilan farq qiluvchi ikki, uch binom ko'paytmasi

$$(x + a_1)(x + a_2) = x^2 + (a_1 + a_2)x + a_1a_2;$$
$$(x + a_1)(x + a_2)(x + a_3) = x^3 + (a_1 + a_2 + a_3)x^2 + (a_1a_2 + a_1a_3 + a_2a_3)x + a_1a_2a_3;$$

shu kabi n ta ko'paytma uchun formula

$$(x + a_1)(x + a_2)\dots(x + a_n) = x^n + S_1x^{n-1} + S_2x^{n-2} + \dots +$$
$$+ S_{n-1}x + S_n;$$

bu yerda:

$$S_1 = a_1 + a_2 + a_3 + \dots + a_n;$$
$$S_2 = a_1a_2 + a_1a_3 + \dots + a_{n-1}a_n;$$
$$S_3 = a_1a_2a_3 + a_1a_2a_4 + \dots + a_{n-2}a_{n-1}a_n;$$
. . . .
$$S_n = a_1a_2a_3 \dots a_n.$$

Agar $a_1 = a_2 = a_3 = \dots = a_n = a$ bo'lsa,

$$(x + a)^n = x^n + S_1x^{n-1}\cdot a + S_2x^{n-2}\cdot a^2 + \dots + S_na^n$$

Gruppalash formulasini hisobga olsak,

$$(x + a)^n = x^n + C_n^1a\cdot x^{n-1} + C_n^2\cdot a^2\cdot x^{n-2} + \dots + a^n.$$

Formulada $a = -a$ desak,

$$(x - a)^n = x^n - C_n ax^{n-1} + C_n^2a^2\cdot x^{n-2} + \dots +$$
$$+ (-1)^kC_n^ka^k\cdot x^{n-k} + \dots +(-1)^na^n.$$

Binom yoyilmasining xossalari:

1. Binom yoyilmasi hadlar soni $(n + 1)$ga teng.
2. Binom yoyilmasi x o'zgaruvchiga nisbatan ko'p had.
3. Binom ko'rsatkichi toq bo'lganda yoyilmada ikkita o'rta had, juft son bo'lganda esa bitta o'rta had bo'ladi.
4. Binom yoyilmasida uning boshidan va oxiridan teng uzoqlikda bo'lgan hadlarining koeffitsiyentlari o'zaro teng.
5. Binom yoyilmasining hamma koeffitsiyentlari yig'indisi 2^n bo'ladi.
6. Binom yoyilmasida toq o'rinda turgan binomial koeffitsiyentlar yig'indisi juft o'rinda turgan binomial koeffitsiyentlar yig'indisiga teng.

2. Kasr funksiya.

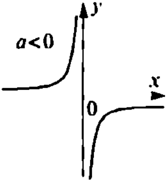
1) $y = \frac{a}{x}$ $x \neq 0$, toq funksiya

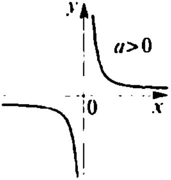
$D(f)$ $\{x; x \in]-\infty; 0[\cup]0; +\infty[\}$

$E(f)$ $\{y; y \in]-\infty; 0[\cup]0; +\infty[\}$

$a < 0 \Rightarrow$ funksiya o'suvchi.

$a > 0 \Rightarrow$ funksiya kamayuvchi.





3. $y = \frac{a}{x^2}$ $a \neq 0$.

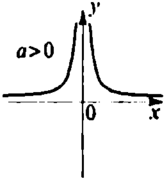
$D(f)$ $\{x; x \in]-\infty; 0[\cup]0; +\infty[\}$;

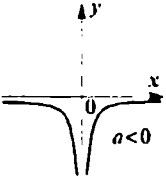
$$E(f)\begin{cases} y \in]0; +\infty[& \text{agar } a > 0 \text{ bo'lsa,} \\ y \in]-\infty; 0[& \text{agar } a < 0 \text{ bo'lsa,} \end{cases}$$

$a > 0, x < 0$ yoki $a < 0$, $x > 0 \Rightarrow$ funksiya o'suvchi,

$a > 0, x > 0$ yoki $a < 0$, $x < 0 \Rightarrow$ funksiya kamayuvchi

juft funksiya





4. Ikkinchi darajali (kvadrat) funksiya.

a) $y = ax^2$, $a \neq 0$, juft funksiya

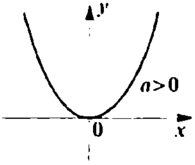
$D(f)$ $\{x;\ x \in]-\infty;\ +\infty[\}$;

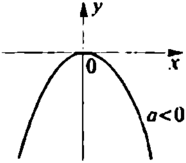
$$E(f)\begin{cases} y \in [0;\ +\infty[& \text{agar } a > 0 \text{ bo'lsa,} \\ y \in]-\infty;\ 0] & \text{agar } a < 0 \text{ bo'lsa,} \end{cases}$$

$a > 0, x > 0$ yoki $a<0$, $x<0 \Rightarrow$ funksiya o'suvchi

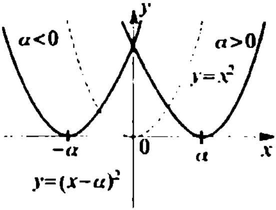
$a > 0, x < 0$ yoki $a<0$, $x>0 \Rightarrow$ funksiya kamayuvchi

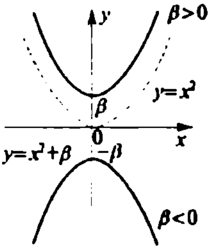
Parabola uchi (0,0) nuqtada, $a > 0$ tarmoqlari yuqori qaragan, $a < 0$ da tarmoqlari pastga qaragan bo'ladi. Funksiya juft.





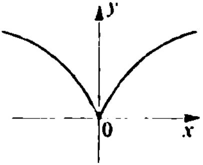
b) $y = ax^2 + bx + c$ funksiyani $y = a(x-\alpha)^2 + \beta$ ko'rinishda yozish mumkin.





Parabola uchi (α, β) nuqta bo'lib, $\alpha = -\frac{b}{2a}$; $\beta = c - \frac{b^2}{4a}$.

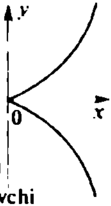
$a > 0$ da tarmoqlari yuqoriga qaragan. $a < 0$ da tarmoqlari pastga qaragan bo'ladi.



5. Ratsional darajali funksiya.

1. $y = \sqrt[3]{x^2} = x^{2/3}$ $D(f)$ $\{x; x \in R\}$ $E(f)\{y; y \in [0; +\infty[\}$

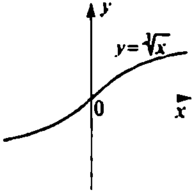
$x < 0 \Rightarrow$ funksiya kamayuvchi, $x > 0 \Rightarrow$ funksiya o'suvchi, juft funksiya.



6. $y = a\sqrt{x^3} = ax^{3/2}$, $a \neq 0$.

$D(f)$ $\{x; x \geq 0\}$,

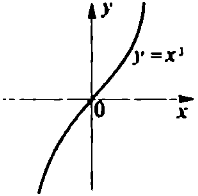
$$E(f)\begin{cases} y \in]0; +\infty[& \text{agar } a > 0 \text{ bo'lsa, o'suvchi} \\ y \in]-\infty; 0[& \text{agar } a < 0 \text{ bo'lsa, kamayuvchi} \end{cases}$$



7. $y = \sqrt[3]{x} = x^{1/3}$ $y = x^{\frac{1}{3}}$

$D(f)$ $\{x; x \in R\}$ $E(f)\{y; y \in R\}$

O'suvchi, toq funksiya

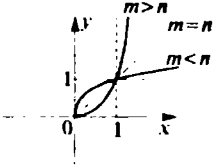


8. $y = x^3$

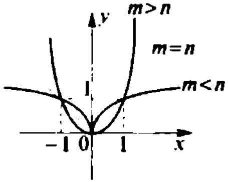
$D(f)$ $\{x; x \in R\}$ $E(f)\{y; y \in R\}$

O'suvchi, toq funksiya

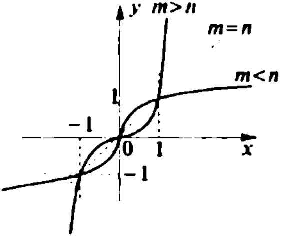
9. $y = x^{\frac{m}{n}}$, $n = 2k$.



10. $y = x^{\frac{m}{n}}$,
$n = 2k+1$
$m = 2p$.



11. $y = x^{\frac{m}{n}}$
$n = 2k + 1$
$m = 2p + 1$



12. Ko'rsatkichli funksiya.

$y = a^x$ $a \neq 1$ $a > 0$ $D(f)$ $\{x; x \in R\}$ $E(f)\{y; y \in]0; +\infty[\}$

$a > 1$ da $a \Rightarrow$ funksiya o'suvchi, $0 < a < 1$ da $\Rightarrow$ funksiya kamayuvchi.

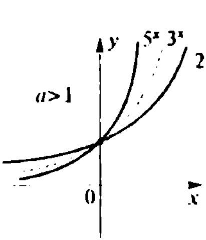
Xossalari

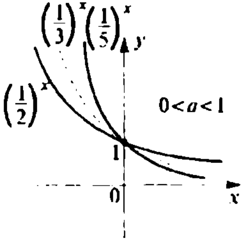
1. $a^0 = 1$ 3. $a^{-x} = \dfrac{1}{a^x}$ 5. $a^{x_1} : a^{x_2} = a^{x_1 - x_2}$

2. $a^x > 0$ 4. $a^{x_1} \cdot a^{x_2} = a^{x_1 + x_2}$ 6. $(ab)^x = a^x \cdot b^x$

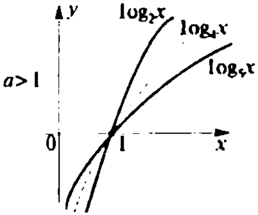
7. $\left(\dfrac{a}{b}\right)^x = \dfrac{a^x}{b^x}$ 8. $a^{x_1} > a^{x_2}$ (<) da: agar $a > 1 \Rightarrow x_1 > x_2$ (<);

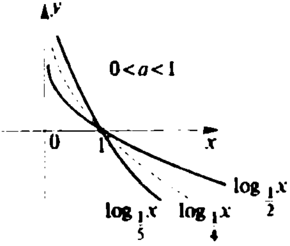
agar $0 < a < 1 \Rightarrow x_1 < x_2$(>)





13. Logarifmik funksiya.
$y = \log_a x$, $a > 0$, $a \neq 1$ $D(f)$ $\{x; x > 0\}$, $E(f)\{y; y \in R\}$
$a > 1 \Rightarrow$ funksiya o'suvchi, $0 < a < 1 \Rightarrow$ funksiya kamayuvchi.

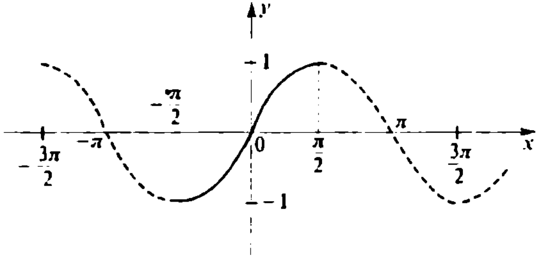




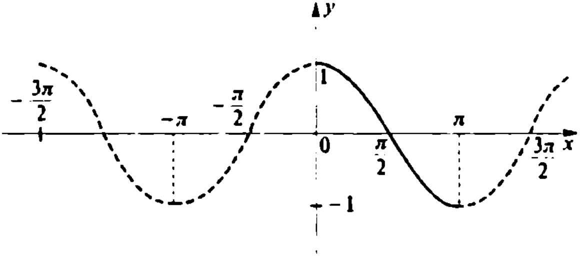
14. Trigonometrik funksiyalar.
$y = \sin x$, $D(f)$ $\{x; x \in R\}$ $E(f)\{y; y \in [-1, 1]\}$
$T = 2\pi$ davriy, toq funksiya

$x \in \left[-\frac{\pi}{2} + 2k\pi;\ \frac{\pi}{2} + 2k\pi\right]$, $k \in z$ — funksiya har bir oraliqda o'suvchi.

$x \in \left[\frac{\pi}{2} + 2k\pi;\ \frac{3\pi}{2} + 2k\pi\right]$, $k \in z$ — funksiya har bir oraliqda kamayuvchi.



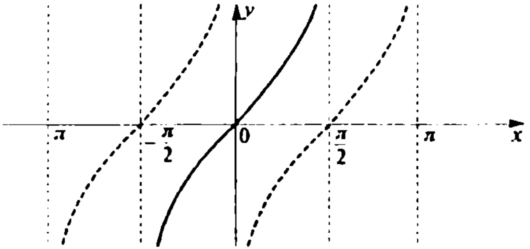
15. $y = \cos x$, $D(f)$ $\{x; x \in R\}$, $E(f)\{y; y \in [-1, 1]\}$
$T = 2\pi$ davriy, juft funksiya
$x \in [2k\pi;\ \pi + 2\pi]$, $k \in z$ — funksiya har bir oraliqda kamayuvchi.
$x \in [\pi + 2\pi;\ 2\pi + 2k\pi]$, $k \in z$ — funksiya har bir oraliqda o'suvchi.



16. $y = \operatorname{tg} x$, $D(f)\ \left\{x; x \in R \setminus \left[\frac{\pi}{2} + k\pi\right]\right\}$, $E(f)\{y; y \in R\}$

$T = \pi$ davriy, toq funksiya.

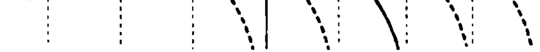
$x \in \left[-\frac{\pi}{2} + k\pi; \frac{\pi}{2} + k\pi\right]$, $k \in z$ — funksiya har bir oraliqda o'suvchi.



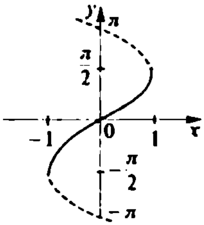
17. $y = \operatorname{ctg} x$, $D(f)\ \{x; x \in]R \setminus k\pi[\}$, $E(f)\{y; y \in R\}$

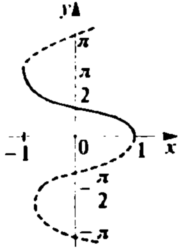
$T = \pi$ davriy, toq funksiya

$x \in]k\pi; \pi + k\pi[$, $k \in z$ — funksiya har bir oraliqda kamayuvchi.



18. $y = \arcsin x$, $D(f)$ $\{x; x \in [-1; 1]\}$. $E(f)\left\{y; y \in \left[-\frac{\pi}{2}; \frac{\pi}{2}\right]\right\}$ funksiya toq o'suvchi.



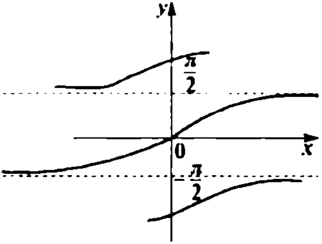


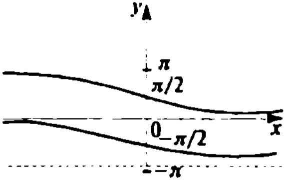
19. $y = \arccos x$, $D(f)$ $\{x; x \in [-1; 1]\}$. $E(f)\{y; y \in [0; \pi]\}$ funksiya kamayuvchi, funksiya juft

20. $y = \operatorname{arctg} x$, $D(f)$ $\{x; x \in R\}$,

$E(f)\left\{y; y \in \left]-\frac{\pi}{2}; \frac{\pi}{2}\right[\right\}$

funksiya toq, o'suvchi.



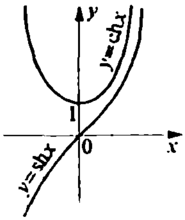


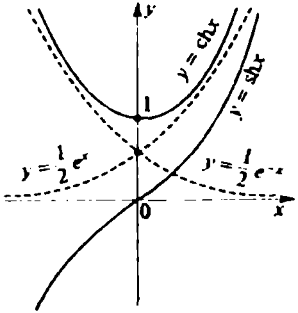
21. $y = \operatorname{arcctg} x$, $D(f)$ $\{x; x \in R\}$, $E(f)$ $\{y; y \in]0; \pi[\}$ funksiya kamayuvchi, funksiya toq.

22. Gipcrbolik funksiyalar.

$y = \text{sh}x = \dfrac{e^x - e^{-x}}{2}$.

$D(f)$ $\{x; x \in R\}$, $E(f)\{y; y \in R\}$ funksiya toq, o'suvchi.





23. $y = \text{ch}x = \dfrac{e^x + e^{-x}}{2}$.

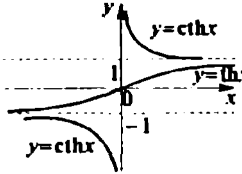
$D(f)$ $\{x; x \in R\}$.

$E(f)\{y; y\in[1; +\infty[\}$ funksiya juft. $x \in]-\infty; 0]$ — kamayuvchi, $x \in [0; +\infty[$ — o'suvchi.

24. $y = \text{th}x = \dfrac{\text{sh}x}{\text{ch}x} = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$, $D(f)$ $\{x; x \in R\}$.

$E(f)$ $\{y; y \in]-1; 1[\}$ — funksiya toq, o'suvchi.

25. $y = \text{cth}x = \dfrac{\text{ch}x}{\text{sh}x} = \dfrac{e^x + e^{-x}}{e^x - e^{-x}}$,



$D(f)$ $\{x; x \in R\setminus\{0\}\}$.
$E(f)$ $\{y; y \in R\setminus[-1; 1]\}$ — funksiya toq, $x \in]-\infty, 0[\cup]0, +\infty[$ — kamayuvchi.

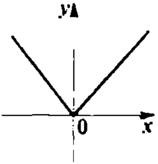
26. Ba'zi bir bo'lakli o'zgarmas funksiya.

1. $y = |x|$

$D(f)$ $\{x;\ x \in R\}$, $E(f)$ $\{y;\ y \in [0;\ +\infty[\}$

$x \in\ ]-\infty;\ 0]$, kamayuvchi

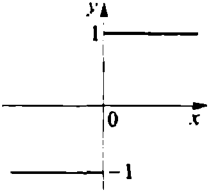
$x \in [0;\ +\infty[$, o'suvchi.



2. $y = \text{sgn}\, x \begin{cases} 1 & x > 0 \\ 0 \quad \text{агар} & x = 0 \quad \text{бўлса} \\ -1 & x < 0 \end{cases}$

$D(f)$ $\{x;\ x \in R\}$, $E(f)$ $\{y;\ -1,\ 0,\ 1\}$

funksiya toq, o'suvchi.



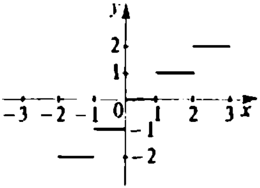
3. $y = [x]$

Ants (butun qism) funksiya.

Agar $x = n + r$, $n \in z$, $0 \leq r < 1$ bo'lsa, $[x] = n$

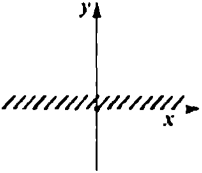
$D(f)$ $\{x;\ x \in R\}$.

$E(f)$ $\{y;\ y \in z\}$ o'suvchi funksiya.



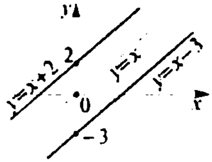
4. $y = \{x\}$

Aniqlanish sohasi $D(f) = R$, qiymatlar sohasi $E(f) = [0;\ 1]$.

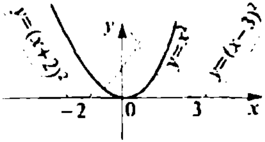


15. Grafikni oddiy almashtirish usullari

1. $y = f(x) + a$. funksiya grafigi ma'lum bo'lgan $f(x)$ funksiya grafigini ordinata o'qi bo'ylab $a > 0$ da a birlik yuqoriga va $a < 0$ bo'lganda a birlik pastga ko'chirish kerak (1-chizma).



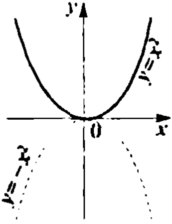
(1-chizma)



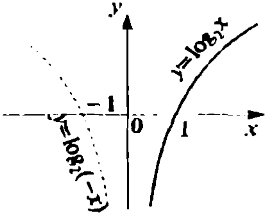
(2-chizma)

2. $y = f(x + a)$ funksiyaning grafigi $a > 0$ bo'lganda $f(x)$ funksiyaning grafigini a birlik abssissa o'qi bo'yicha chapga, $a < 0$ bo'lganda esa a birlik o'ngga ko'chirish kerak (2-chizma).

3. $y = -f(x)$, funksiyaning grafigi $y = f(x)$ funksiya grafigining abssissa o'qi bo'ylab simmetrik tasviri yasaladi. (3-chizma).



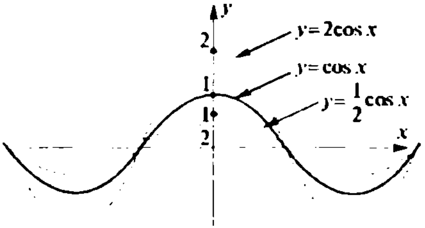
(3-chizma)



(4-chizma)

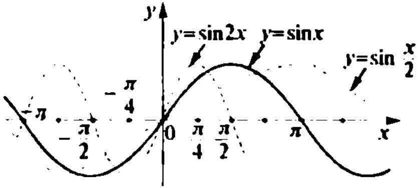
4. $y = f(-x)$ funksiyaning grafigini yasash uchun $y = f(x)$ funksiyaning grafigini ordinata o'qi bo'ylab simmetrik tasviri yasaladi. (4-chizma).

5. $y = Af(x)$. funksiya grafigi $A > 1$ da $y = f(x)$ funksiya grafigining ordinatasini A marta kattalashtiriladi. $0 < A < 1$ da $y = f(x)$ funksiya grafigining ordinatasini $\frac{1}{A}$ marta kichiklashtirish kerak (5-chizma).



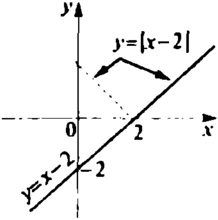
(5-chizma)

6. $y = f(kx)$ funksiyaning grafigini yasash uchun $k > 1$ da $y = f(x)$ funksiya grafigi abssissa o'qi bo'ylab k marta siqiladi. $0 < k < 1$ da $y = f(x)$ funksiya grafigi abssissa o'qi bo'ylab $\frac{1}{k}$ marta kengaytiriladi (6-chizma).

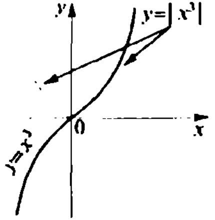


(6-chizma)

7. $y=|f(x)|$ funksiyaning grafigini yasash uchun $y=f(x)$ funksiya grafigining $f(x)\geq 0$ qiymatlardagi grafigini o'zgarishsiz qoldiradi $f(x)<0$ dagi grafigini absissa o'qi bo'yicha simmetrik tasvirini yasash kerak (7-chizma).



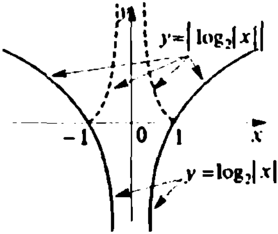
(7-chizma)



(8-chizma)

8. $y=f(|x|)$ funksiyaning grafigini yasash uchun birinchidan $y=f(x)$ funksiya grafigining $f(x)\geq 0$ dagi grafigini yasash kerak, keyin hosil bo'lgan funksiya grafigini ordinata o'qi bo'yicha simmetrik tasvirini yasash kerak (8-chizma).

9. $y=|f(|x|)|$ funksiya grafigini yasash uchun $y=f(|x|)$ grafigini yasab, $f(x)<0$ qiymatlardagi grafigini abssissa o'qiga nisbatan simmetrik tasvirini yasash kerak (9-chizma).



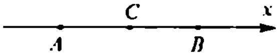
(9-chizma)

Eslatma: Ba'zi bir funksiyalarning grafigini yasashda (almashtirish usuli bilan) yuqorida ko'rsatilgan usullarning bir nechtasini ketma-ket qo'llab yasaladi. Masalan, $y = 4x - 2x^2 + 3$ funksiya grafigini yasash uchun kvadrat uch had funksiyani $y = -2(1 - x)^2 + 5$ ko'rinishda yozish mumkinligidan, parabola grafigini yasash quyidagicha bajariladi:

$y_1 = x^2$ grafigi yordamida $y_2 = (-x)^2$ funksiya grafigi yasaladi, keyin almashtirish usulida $y_3 = (-x + 1)^2$ grafigi, undan keyin $y_4 = -2(-x + 1)^2$ yasaladi. Oxirida bu funksiya grafigi yordamida $y = -2(-x + 1)^2 + 5 = 4x - 2x^2 + 3$ funksiya grafigi yasaladi.

16. To'g'ri burchakli koordinatalar sistemasi

1. O'qdagi koordinatalar sistemasida $A(x_1)$ va $B(x_2)$ nuqtalar koordinatalarda berilgan bo'lsin. AB kesma uzunligi



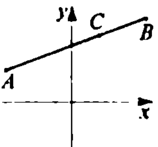
$$|AB| = |x_1 - x_2|$$

AB kesmani berilgan λ nisbatda $(0 < \lambda \leq 1)$ bo'luvchi $\frac{AC}{CB} = \lambda$ C nuqtaning koordinatasi

$$x_c = \frac{x_1 + \lambda x_2}{1 + \lambda},$$

$\lambda = 1$ da $x_c = \frac{x_1 + x_2}{2}$ bo'lib, kesma teng ikkiga bo'linadi.

2. To'g'ri burchakli (Dekart) koordinatalar sistemasida $A(x_1; y_1)$ va $B(x_2; y_2)$ nuqtalar berilgan bo'lsin.



1) AB kesma uzunligi

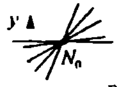
$$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}, \quad (1)$$

2) AB kesmani $\frac{AC}{CB} = \lambda$ nisbatda $(0 < \lambda \leq 1)$, bo'luvchi C nuqtaning koordinatalari:

$$x_c = \frac{x_1 + \lambda x_2}{1 + \lambda}, \quad y_c = \frac{y_1 + \lambda y_2}{1 + \lambda}. \qquad (2)$$

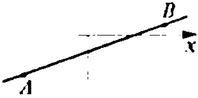
$\lambda = 1$ da kesma teng ikkiga bo'linadi.

3) $N_0(x_1, y_1)$ nuqtadan o'tuvchi to'g'ri chiziq tenglamasi:



$$y - y_1 = k(x - x_1). \qquad (3)$$

4) Berilgan $A(x_1; y_1)$ va $B(x_2; y_2)$ nuqtalardan o'tuvchi (bu bitta bo'ladi) to'g'ri chiziq tenglamasi:



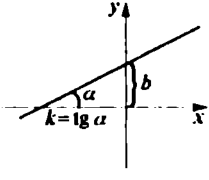
$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1}. \qquad (4)$$

5) To'g'ri chiziqning umumiy tenglamasi

$$Ax + By + C = 0, \qquad (5)$$

ko'rinishda bo'lib, $A = 0$ da to'g'ri chiziq $0x$ o'qqa parallel, $B = 0$ da to'g'ri chiziq $0y$ o'qqa parallel, $C = 0$ da esa to'g'ri chiziq koordinata boshidan o'chadi. $A = C = 0$ da to'g'ri chiziq $0x$ o'q bilan ustma-ust tushadi, $B = C = 0$ da to'g'ri chiziq $0y$ o'q bilan ustma-ust tushadi.

6) $y = kx + b$, (6) to'g'ri chiziqning burchak koeffitsiyentli tenglamasi.



7) $N_1(x_1; y_1)$ nuqtadan o'tib $y = k_1x + b_1$ to'g'ri chiziqqa parallel to'g'ri chiziq tenglamasi

$$y - y_1 = k_1(x - x_1),$$

perpendikular tenglamasi esa

$$k_1(y - y_1) = -(x - x_1).$$

8) $y = k_1x + b_1$ va $y = k_2x + b_2$ to'g'ri chiziqlar orasidagi burchak

$$\operatorname{tg} \alpha = \frac{k_1 - k_2}{1 + k_1k_2}. \tag{7}$$

$k_1 = k_2$, to'g'ri chiziqlarning parallellik sharti, $1 + k_1k_2 = 0$, to'g'ri chiziqlarning perpendikularlik sharti.

9) $N_1(x_1; y_1)$ nuqtadan $A_1x + B_1y + C_1 = 0$ to'g'ri chiziqqacha bo'lgan masofa

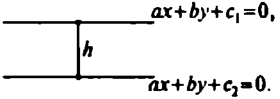
$$d = \left|\frac{A_1x_1 + B_1y_1 + C_1}{\sqrt{A_1^2 + B_1^2}}\right|. \tag{8}$$

10) $A(x_1; y_1)$, $B(x_2; y_2)$ va $C(x_3; y_3)$ nuqtalarning bir to'g'ri chiziqda yotish sharti

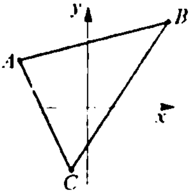
$$\frac{y_3 - y_1}{y_2 - y_1} = \frac{x_3 - x_1}{x_2 - x_1}. \tag{9}$$

$ax + by + c_1 = 0$ da $ax + by + c_2 = 0$ parallel to'g'ri chiziqlar orasidagi masofa

$$h = \frac{|c_1 - c_2|}{\sqrt{a^2 + b^2}}.$$



11) Agar A, B va C nuqtalar uchun (9) shart bajarilmasa, bu nuqtalardan uchburchak yasash mumkin:



$A(x_1; y_1)$, $B(x_2; y_2)$ va $C(x_3; y_3)$ nuqtalardan yasalgan uchburchakda:

1) uchburchak tomonlar tenglamasi (4) formuladan topiladi;

2) uchburchak tomonlar uzunligi (1) formula yordamida topiladi;

3) balandlik, mediana, bessiktrisa tenglamalarini topishga (2), (3) va (4), (7) formulalar yordam beradi.

4) uchburchak yuzi

$$S = \pm\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \pm\frac{1}{2}\left[\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix} + \begin{vmatrix} x_2 & y_2 \\ x_3 & y_3 \end{vmatrix} + \begin{vmatrix} x_3 & y_3 \\ x_1 & y_1 \end{vmatrix}\right]$$

ishorani tanlash | | ifoda ishorasi bilan bir xil olinadi.

17. Ikkinchi tartibli chiziq

Ikkinchi tartibli chiziqning umumiy tenglamasi

$$a_{11}x^2 + 2a_{12}xy + a_{22}y^2 + 2a_1x + 2a_2y + a_3 = 0,$$

ko'rinishda bo'lib, bunda:

1) $a_{12} = 0$, $a_{11} = a_{22} \neq 0 \Rightarrow$ ikkinchi tartibli aylanani ifodalaydi;

2) $a_{11}^2 + a_{12}^2 + a_{22}^2 > 0$, bo'lsa, ikkinchi tartibli chiziq quyidagi kanonik (sodda) ko'rinishlarning biriga keladi:

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ — ellips; $\frac{x^2}{a^2} + \frac{y^2}{b^2} = -1$ — (mavhum ellips); $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ — giperbola; $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 0$ — kesishuvchi ikkita to'g'ri chiziq; $y^2 = 2px$ — parabola.

$x^2 = a^2 (a \neq 0)$ — ikki parallel to'g'ri chiziq,
$x^2 = -a^2 (a \neq 0)$ — ikki mavhum parallel chiziq,
$x^2 = 0$ — ikki ustma-ust tushgan to'g'ri chiziq.

Ikkinchi tartibli chiziq invariant klassifikasiyasida

$$I = a_{11} + a_{22},\quad \nabla_2 = \begin{vmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{vmatrix},\quad \nabla_3 = \begin{vmatrix} a_{11} & a_{12} & a_1 \\ a_{12} & a_{22} & a_2 \\ a_1 & a_2 & a_3 \end{vmatrix}$$

1) $\nabla_2 \neq 0 \Rightarrow$ ikkinchi tartibli chiziq bitta simmetriya markaziga ega;

2) $\nabla_2 = 0,\ \nabla_3 \neq 0 \Rightarrow$ ikkinchi tartibli chiziq simmetriya markaziga ega emas;

3) $\nabla_2 > 0,\ \angle \cdot \nabla_3 < 0 \Rightarrow$ ikkinchi tartibli chiziq ellips;

4) $\nabla_2 < 0,\ \nabla_3 \neq 0 \Rightarrow$ ikkinchi tartibli chiziq giperbola;

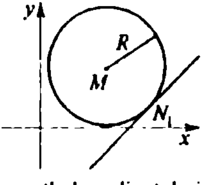
5) $\nabla_2 = 0,\ \nabla_3 \neq 0 \Rightarrow$ ikkinchi tartibli chiziq porabola.

Ikkinchi tartibli chiziqlarda xususiy hollar:

1. Aylana

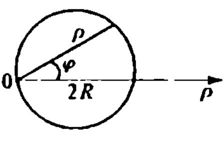
Markazi $M(a;\ b)$ nuqtada radiusi R, bo'lgan aylananing kanonik (sodda) tenglamasi:

$$(x - a)^2 + (y - b)^2 = R^2.$$



Parametrik tenglamasi:

$\{x = a + R\cos t,\ y = b + R\sin t\}$ $a,\ b \in R$ $t \in [0, 2\pi]$



Aylananing qutb koordinatalari tenglamasi: $\rho = 2R\cos\varphi$.

$N_1(x_1, y_1)$ nuqtadagi urinma tenglamasi.

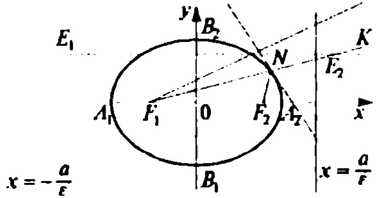
$(x-a)(x_1-a)+(y-b)(y_1-b)=R^2$ — aylana yopiq egri chiziq. Aylana uzunligi $C=2\pi R$. Aylana bilan chegaralangan doira yuzi $S=\pi R^2$.

2. Ellips

Dekart koordinatalar sistemasidagi koordinata o'qlariga simmetrik ellips kanonik (sodda) tenglamasi.

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ $(a>b)$ $|AA_1|=2a$ — katta o'qi, $|BB_1|=2b$ — kichik o'qi,

A, A_1, B, B_1 — ellips o'qlari, $O(0;0)$ — simmetriya markazi, $c^2=a^2-b^2$



$F_1(-C;0)$, $F_2(C;0)$ — fokus, $r_1=|F_1N|=a+\varepsilon x$, $r_2=a-\varepsilon x$ — fokal radius

$\varepsilon=\frac{c}{a}<1$ — ekssentrisitet, $x=-\frac{a}{\varepsilon}$, $x=\frac{a}{\varepsilon}$ direktrisa tenglamasi

$N_1(x_1, y_1)$ nuqtadagi urinma tenglamasi $\frac{xx_1}{a^2}+\frac{yy_1}{b^2}=1$,

normal tenglamasi

$$y-y_1=\frac{a^2y_1}{b^2x_1}(x-x_1).$$

Ellipsda: $\frac{r_1}{a_1} = \varepsilon$ yoki $\frac{r_2}{a_2} = \varepsilon$, $r_1 + r_2 = 2a$, $|NE_2| = a_2$, $|NE_1| = a_1$.

N nuqtadagi urinma uchun: $F_2\widehat{NL} = \widehat{LNK}$.

Ellipsning parametrik tenglamasi $\begin{cases} x = a\cos t, \\ y = b\sin t. \end{cases}$

Qutb koordinatalar sistemasidagi tenglamasi:

$$\rho = \frac{b^2}{a(1-\varepsilon\cos\varphi)}.$$

Ellips bilan chegaralangan yuza: $S = \pi ab$.

Ellips yopiq egri chiziq bo'lib, $a = b$ da aylana bo'lib, $\frac{b}{a} < 1$ bo'lsa, aylananing qisilishi $\frac{b}{a} > 1$ bo'lsa, aylananing cho'zilishi bo'lib, fokus katta o'qda bo'ladi.

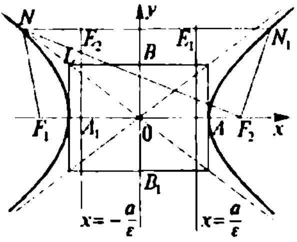
3. *Giperbola*

Dekart koordinatalar sistemasida fokusi Ox o'qida bo'lgan giperbolaning kanonik (sodda) tenglamasi $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ $(a > b)$, $|AA_1| = 2a$ — haqiqiy o'qi $|BB_1| = 2b$ — mavhum o'q.

$O(0;\ 0)$ — simmetriya markazi $A_1(-a;\ 0)$, $A(a;\ 0)$ — uchi $c^2 = a^2 + b^2$.

$F_1(-C;\ 0)$, $F_2(C;\ 0)$ — fokusi, $r_1 = |F_1N| = -a - \varepsilon x$, $|F_2N| = r_2 = a - \varepsilon x$ — fokal radius,

$\varepsilon = \frac{c}{a} > 1$ — ekssentrisasi.



$x = -\frac{a}{\varepsilon}$, $x = \frac{a}{\varepsilon}$ — direktrisa tenglamasi.

Giperbolaning ixtiyoriy $N_1(x_1 y_1)$ nuqtasida urinma tenglamasi: $\frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1$, normal tenglamasi: $y - y_1 = -\frac{a^2 y_1}{b^2 x_1}(x - x_1)$.

Assimtota tenglamalari: $y = \frac{b}{a}x$, $y = -\frac{b}{a}x$.

Giperbolada: $\forall N$ nuqtadagi urinma uchun: $F_2\widehat{NL} = F_1\widehat{NL}$.

Giperbolaning parametrik tenglamasi: $\begin{cases} x = a\,\text{cht}, \\ y = b\,\text{sht}. \end{cases}$

Qutb koordinatalar sistemasidagi tenglamasi:

$$\rho = \frac{b^2}{a(1-\varepsilon\cos\varphi)}.$$

Giperbola koordinata o'qlariga simmetrik bo'lib cheksizlikka qarab ketuvchi o'ng va chap tarmoqlardan iborat. ε — ortishida giperbola tarmoqlari «kengayadi». $b > a$ da $\frac{x^2}{b^2} - \frac{y^2}{a^2} = 1$ qo'shma giperbola asimptotalari $y = \pm\frac{b}{a}x$ fokuslari $F_1(0; -c)$ va $F_2(0; c)$ nuqtalarda bo'lib, $\varepsilon = \frac{c}{b} > 1$ $a = b$ da teng tomonli giperbola $x^2 - y^2 = a^2$ bo'lib, $\varepsilon = \sqrt{2}$.

4. Parabola

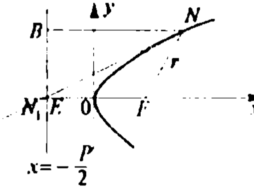
Dekart koordinatalar sistemasida Ox o'qqa simmetrik parabolaning kanonik (sodda) tenglamasi $y^2 = 2px$ $O(0; 0)$ — uchi

$|EF| = P$, $F\left(\frac{P}{2}; 0\right)$ — fokus,

$\varepsilon = \dfrac{|FN|}{NB} = 1$ — ekssentrisasi.

$x = -\dfrac{p}{2}$ — direktrisa tenglamasi.

$R = |FN| = x + \dfrac{p}{2}$ — fokal radius.



$N(x_1, y_1)$ nuqtadagi urinma tenglamasi:

$yy_1 = p(x + x_1)$, normal tenglamasi $y - y_1 = \dfrac{y_1}{p}(x - x_1)$.

Parabolaning N nuqtasiga o'tkazilgan urinma uchun: $\forall N$ да $\widehat{FNN_1} = \widehat{FN_1N}$.

Parametrik ko'rinishdagi tenglama: $\begin{cases} x = \dfrac{t^2}{2p}, \\ y = t. \end{cases}$

Qutb koordinatalar sistemasidagi tenglamasi:

$$\rho = \frac{p}{1 - \cos\varphi}.$$

Parabola koordinata o'qlariga simmetrik bo'lib cheksizlikka qarab ketuvchi bitta tarmoqdan iborat.

Parabolaning boshqa tenglamalari.